Heat and Mass Transfer Transients in Cylinder Drying:

Part I.
Unfelted Cylinders

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An investigation has been made of transient heat transfer and water removal on an unfelted cylinder dryer. This investigation has included the development of a theory for describing conduction of heat in the drying material and an experimental testing of the adequacy of the theory.

The theory describes the heat transfer and evaporation of water in terms of a second-order partial-differential equation and appropriate boundary conditions. Numerical solutions obtained on a digital computer are presented.

The experimental work, performed on a specially constructed laboratory dryer, included measurements of temperatures at internal points in a drying sheet and also measurements of water removed during drying.

Good agreement was found between theory and experiment, and the usefulness of the theory is demonstrated in the analysis of water removal in some drying experiments.

Although the work was primarily concerned with a description of the paper drying process, the methods should apply equally well to the drying of other materials on heated cylinders.

THEORY

Heated cylinders have been utilized for drying of sheet materials and slurries for at least a century. The paper and textile industries have made the largest use of this method of drying, owing primarily to the convenience of handling these materials on rotating cylinders. The location of drying cylinders on a paper making machine is schematically indicated in Figure 1.

Although cylinder, or drum, drying is an old process for which the industries have a great deal of know how gained from experience, there has not yet been a detailed description of the heat transfer controlling the process. Such a description is of more than academic interest in analyzing the factors which govern the drying rate and also in calculating the temperatures attained by a material in various stages of the drying process. The interest in obtaining a more detailed understanding of this drying process has been demonstrated by reports of a number of researchers on this topic in recent years.

Attwood and Smith (*1*) simulated the process of drying on many cylinders in series and so were able to make an experimental study of the effects of varying different machine conditions. Dreshfield and Han (*2*, *3*), employing a beta-ray transmission technique together with dye migration, studied the movement and distribution of moisture in a wet sheet contacted with a hot surface. Han and Ulmanen (*4*) extended this work, obtaining both moisture and temperature measurements. On the basis of these measurements they were able to evaluate an apparent thermal conductivity for the wet sheet and to relate the conductivity to the moisture content and/or apparent density of the sheet.

Brauns and Janson (*5*) and Janson and Nordgren (*6*) have reported extensive experimental studies of heat transfer on a Yankee dryer. These studies included measurements of temperatures through the thickness of the dryer shell, in the sheet, and in the dryer felt. On the basis of these measurements some estimates were made of various heat transfer coefficients and evaporation rates.

While these studies have contributed much to the understanding of cylinder drying, it remains difficult to interpret their significance without a quantitative theory. The first part of this paper presents a theory describing the heat transfer, of which the evaporation is a part. The basis of this approach has been published earlier and demonstrated in the calculation of drying rates on industrial machines (*7*, *8*). This presentation includes fewer assumptions, and more precise and detailed solutions are obtained.

Development of Cylinder-Drying Equations

When a wet material is put in contact with the hot surface of a drying cylinder it will absorb heat from the cylinder, and evaporation will be induced at any exposed surface. The description of temperature rise in the material is a transient heat transfer problem. However when the process has been in operation for some time, and conditions maintained constant, a steady state will exist at any point. When one applies either the Eulerian approach to a stationary point in the traverse of the sheet, or the Lagrangian approach to a point moving with the sheet, the usual heat balance methods will yield the following equations:

$$k_x \frac{\partial^2 t}{\partial x^2} + k_y \frac{\partial^2 t}{\partial y^2} + k_z \frac{\partial^2 t}{\partial z^2} - V\rho c \frac{\partial t}{\partial z} = 0 \qquad (1)$$

The last term is due to the motion of the sheet, which has been taken in the *z* direction. This is the general partial differential equation describing heat transfer in cylinder drying. It involves only the assumptions that steady state has been achieved for the over-all process and that the controlling mechanism for transfer of heat through the sheet is conduction.

By neglecting edge effects and assuming the conductivity to be equal in all directions one can reduce the equation to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial z^2} - \frac{V}{\alpha} \frac{\partial t}{\partial z} = 0 \qquad (2)$$

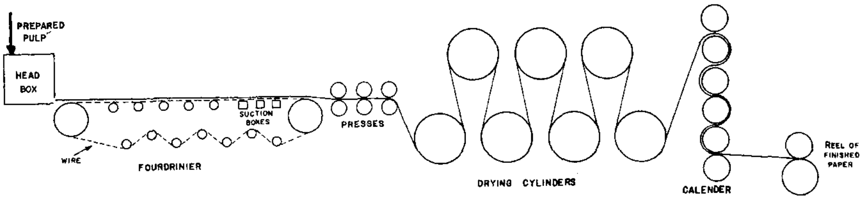

Fig. 1. Schematic sketch of paper making machine.

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When one realizes that position z is directly related to time through the velocity

$$V = \frac{dz}{d\theta}$$

it is possible to eliminate z in terms of θ by making the substitutions

$$\frac{\partial t}{\partial z} = \frac{1}{V}\frac{\partial t}{\partial \theta}$$

and

$$\frac{\partial^2 t}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 t}{\partial \theta^2}$$

The result of these substitutions into Equation (2) is

$$\frac{\partial^2 t}{\partial x^2} + \frac{1}{V^2}\frac{\partial^2 t}{\partial \theta^2} - \frac{1}{\alpha}\frac{\partial t}{\partial \theta} = 0 \quad (3)$$

A study of this equation in a little more detail indicates a valuable simplification which may be applied. Consider the problem defined by Equation (3) and these boundary conditions:

$t = t_o$ (a constant) for $\theta = 0$

$t = t_s$ (a constant) for $x = 0$

$\frac{\partial t}{\partial x} = 0$ for $x = X$

The complete solution to this problem as obtained by the Fourier series technique is

$$t = t_s - \frac{4(t_s - t_o)}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}$$

$$\sin\frac{(2n+1)\pi x}{2X}\exp\left[V^2/2\alpha\left(1 - \sqrt{1 + \frac{\alpha^2\pi^2(2n+1)^2}{X^2V^2}}\right)\theta\right]$$

If $\alpha^2(2n+1)^2\pi^2/X^2V^2$ is much smaller than 1, then the terms under the radical can be set equivalent to

$$\left(1 + \frac{\alpha^2\pi^3(2n+1)^2}{2X^2V^2}\right)^2$$

and the solution reduced to

$$t = t_s - \frac{4(t_s - t_o)}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}$$

$$\sin\frac{(2n+1)\pi x}{2X}\exp\left(-\frac{\alpha\pi^2(2n+1)^2}{4X^2}\theta\right)$$

This reduced solution is exactly what would result from employing the simpler differential equation

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\frac{\partial t}{\partial \theta}$$

in place of Equation (3).

The boundary conditions employed in this problem are considerably simpler but not radically different from those which arise from the drying problem. The requirement for obtaining the simplified solution

$$\frac{\alpha^2\pi^2(2n+1)^2}{X^2V^2} << 1$$

can be shown to be valid for conditions obtaining on a cylinder dryer (7). This analysis then permits the simplification of describing the drying problem with the reduced equation

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\frac{\partial t}{\partial \theta} \quad (4)$$

It will be noted that this step has also served to eliminate the machine speed, V, as a parameter in the equation.

Boundary Conditions

The mathematical boundary conditions to be observed in solving Equation (4) are determined by the physical conditions existing on the machine. These conditions may vary from place to place on a machine composed of many cylinders, and even on a given cylinder there may be major differences, particularly if a dryer felt is present. For this reason it is convenient to consider a dryer in sections or phases as depicted in Figure 2. It is seen from Figure 2 that the phases are defined by location, on a cylinder (I, II and III) or off (IV) a cylinder, and by the presence (II) or absence (I and III) of a dryer felt covering. This division is convenient in that the problem may be generalized for each phase. Then, when solutions are obtained, they may be applied to successive phases to describe events on a dryer containing any number or sequence of phases and cylinders.

When one considers the start of a phase as time zero, the initial condition to be observed is given by the temperature distribution through the thickness of the sheet at time zero:

$$t = f(x) \quad \theta = 0 \quad (5)$$

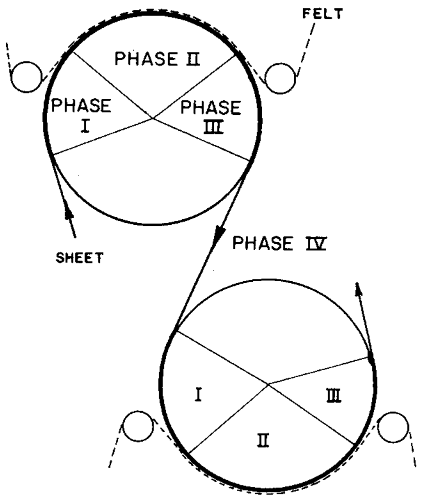

Fig. 2. Phases on a cylinder dryer.

The two additional restrictions required to complete the formulation of the heat transfer are determined by the heat fluxes at the surfaces:

$$-k\left(\frac{\partial t}{\partial x}\right)_{x=0} = Q_{x=0} \quad (6)$$

$$-k\left(\frac{\partial t}{\partial x}\right)_{x=X} = Q_{x=X} \quad (7)$$

The three conditions (5), (6), and (7) together with Equation (4) complete the general description of heat transfer in a material on a hot drying cylinder. Before a solution can be attempted it is necessary to describe the fluxes in terms of the surface temperatures.

Heat may enter or leave a boundary surface by convection, radiation, or evaporation. In drying, radiation is negligible compared with convection and evaporation; hence $Q = q$ conv. $+ q$ evap.

Considering phases I, II, and III one can write for the surface adjacent to the cylinder (where no moisture can be removed by evaporation)

$$Q_{x=0} = H_1(t_s - t_{x=0})$$

At the exposed surface there may be evaporation, and hence in phases I and III

$$Q_{x=X} = H_2(t_{x=X} - t_a) + \lambda\left(\frac{dw}{d\theta}\right)_{x=X}$$

In phase II

$$Q_{x=X} = H_2(t_{x=X} - t_f) + \lambda\left(\frac{dw}{d\theta}\right)_{x=X}$$

In phase IV there may be evaporation at both faces, and

$$Q_{x=0} = H_1(t_a - t_{x=0})\ \lambda\left(\frac{dw}{d\theta}\right)_{x=0}$$

$$Q_{x=X} = H_2(t_{x=X} - t_a) + \lambda\left(\frac{dw}{d\vartheta}\right)_{x=X}$$

It is further required to relate the evaporation rates to surface temperatures. This may be done in the usual manner with a mass transfer coefficient and the vapor pressures:

$$\lambda\left(\frac{dw}{d\theta}\right)_{x=0} = H_{1m}(y_o - y_a)$$

The number of separate parameters appearing in the problem can be reduced by substituting the dimensionless quantities given below and by relating heat and mass transfer coefficients through the j-factor analogy of Chilton and Colburn (9).

Defining the dimensionless quantities

$$r = \frac{x}{X}$$

TABLE 1. SUMMARY OF CYLINDER DRYING PROBLEM

differential equation

$$\frac{\partial^2 t}{\partial r^2} = \frac{\partial t}{\partial \tau}$$

initial conditions:

$$t = f_{(r)} \quad \tau = 0$$

boundary conditions:

Phase I and III

$$\left(\frac{\partial t}{\partial r}\right)_{r=0} = -R_1(t_s - t_{r=0})$$

$$\left(\frac{\partial t}{\partial r}\right)_{r=1} = -R_2(t_{r=1} - t_a) - 4.65R_2(y_1 - y_a)\, t_b$$

Phase II

$$\left(\frac{\partial t}{\partial r}\right)_{r=0} = -R_1(t_s - t_{r=0})$$

$$\left(\frac{\partial t}{\partial r}\right)_{r=1} = -R_2(t_{r=1} - t_f) - 4.65R_2(y_1 - y_f)\, t_b$$

Phase IV

$$\left(\frac{\partial t}{\partial r}\right)_{r=0} = -R_1(t_a - t_{r=0}) + 4.65R_2(y_o - y_a)\, t_b$$

$$\left(\frac{\partial t}{\partial r}\right)_{r=1} = -R_2(t_{r=1} - t_a) - 4.65R_2(y_1 - y_a)\, t_b$$

$$\tau = \frac{\alpha\theta}{X^2}$$

$$R_1 = \frac{H_1 X}{k}$$

$$R_2 = \frac{H_2 X}{k}$$

and relating the heat and mass transfer coefficients by the *j* factor, one may rewrite the equations as summarized in Table 1.

The constant, $4.65t_b$, in the boundary-conditions equation (Table 1) comes from the evaluation of properties of the air stream, which appear in the *j* factor relations.

Before an analytical solution to this problem is attempted, it would be necessary to express the mole fractions y_1 and y_o as definite functions of the temperatures. This could probably be done quite satisfactorily in terms of logarithm of vapor pressure and inverse temperature. However the resulting complex boundary conditions define a problem which is not readily, if at all, solvable by the usual analytical techniques. Hence it was decided to obtain solutions by a numerical method, for which it is more convenient not to substitute for the mole fractions.

Numerical Solutions

Numerical solutions to the cylinder drying problem were obtained by the method of finite difference approximations. This method has been described in many mathematics texts, including Mickley, Sherwood, and Reed's on chemical engineering mathematics (*10*).

The final equations, solved for the desired temperatures, are summarized in Table 2. Actually the boundary conditions equations are not explicit in that the mole fractions are functions of temperature. In the numerical work however the problem of solving for the temperature in an implicit equation was avoided by taking the value of y_1 or y_o at the preceding time $(\tau - \Delta\tau)$ rather than at time τ. While not rigorously correct, this procedure was justified on the basis that small increments were used in the calculations, within which y did not change greatly. In fact within the desired precision this method of calculation introduced no error.

The equations as written in Table 2 may be used with any absolute temperature scale. To use Fahrenheit or Centigrade temperatures it is only necessary to replace t_b with $(t_b + 460)$ or $(t_b + 273)$.

In the development of these equations evaporation has been treated as occurring from a plane water surface (constant rate period). Adjustments must therefore be made in the evaporation heat transfer term when the sheet becomes dryer (falling rate period). It is noted that the wet-bulb temperature is the natural lower limit on temperatures calculated from the equations for phase IV.

The quantity W_T in Table 2 is a dimensionless group which evaluates the evaporation occurring within a phase. It is by definition

$$W_T = \frac{\alpha \lambda w}{X k t_b}$$

Hence the equation for W_T provides a means of calculating evaporation simultaneously with the temperature calculations.

To complete the solutions it remains to show that this set of finite difference equations is stable and that they will converge to the exact solution when the increments are taken sufficiently small. The convergence of a finite difference solution is guaranteed, in general, if the exact boundary value problem possesses a unique solution. In this case a physical situation has been described for which a unique solution must exist.

Fulfillment of the stability criterion is a necessary part of the numerical evaluation. In this case the solution was found to be stable provided that the modulus $[(\Delta r)^2/\Delta\tau - 2]$ was not less than zero.

Application of Numerical Solutions

In the equations of Table 2 there are five independent parameters; R_1, R_2, t_s, t_a, and y_a. In phase II the felt surface temperature is also a parameter. Values must be specified for each of these in

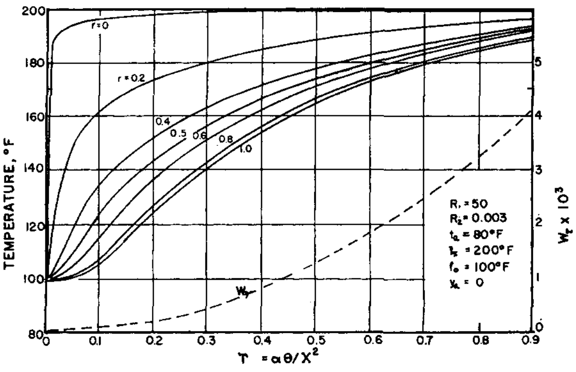

Fig. 3. Example of numerical solution for phase 1.

making calculations, but these values need not remain constant even within a single phase of the drying cycle. Once values for the parameters have been chosen, the calculation of sheet temperatures from the equations is straightforward. However the time increments used must be very small in order to obtain stable and accurate results. In the calculations made for this study the sheet was divided into 10 slices ($\Delta r = 0.1$), and $\Delta\tau = 0.0025$. For a wet sheet 0.010 in. thick this would be equivalent to a time increment of about 0.002 sec. As a consequence the number of separate calculations required to describe the temperature distribution, and to evaluate the evaporation, were too numerous to be done by hand or even with a desk machine, and the use of a high-speed computer was dictated.

To demonstrate the nature of the solutions, and in conjunction with the experimental study, the equations for phase I and phase IV were programmed on an IBM 650 digital computer. For this program increments were chosen for $\Delta r = 0.1$, and $\Delta\tau = 0.0025$, giving a modulus $[(\Delta r)^2/\Delta\tau - 2]$ of 2. The program, for calculations on the IBM 650, was written with optimized instruction location. Machine time required for calculation of temperatures for a range of τ from $0 \rightarrow 1$ was 5 min.

An example of the results obtained with one set of parameter values (all constant) is given in Figure 3. Additional examples and details of the solution and computer program are given in a thesis by Hansen (*11*).

With a complete program for calculation of all four phases, one could treat an entire dryer composed of any number of cylinders and phases. The primary difficulty would be assigning values to the different parameters, particularly the heat transfer coefficient between cylinder and sheet, and, in phase II, the felt temperature.

EXPERIMENTAL CONFIRMATION ON AN UNFELTED CYLINDER

Description of Experimental Apparatus

The experimental study was performed on a single cylinder laboratory dryer constructed specifically for this purpose. For full details see reference *11*.

The drying cylinder was machined from a section of 12-in. mild steel pipe and measured 12.75 in. O.D. and 14 in. in width. The shell thickness was ⅜ inch. It was driven by a ¼-hp. motor through a gear reducer and step pulley. By changing pulleys, cylinder speeds from 5 to 75 rev./min. were obtained. A blower and duct over the cylinder allowed independent control of relative air speed during drying.

The samples material passed back and forth between reels on both sides of the cylinder. (The cylinder could be driven in either direction.) Weights attached to these reels controlled tension on the sample. Heating was accomplished by boiling water, or ethylene glycol solution, inside the cylinder with an immersion heating coil. Vapors passed to an external condenser, and the condensate returned to the cylinder.

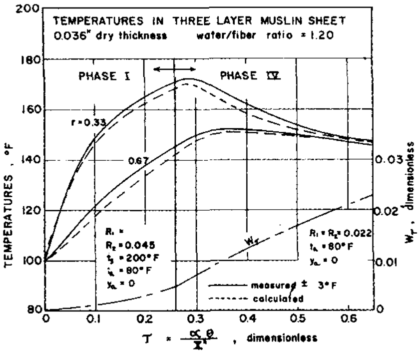

Fig. 4. Comparison of measured and calculated temperatures.

The temperature of the cylinder shell was measured with iron-constantan thermocouples embedded in the shell at the external surface. These thermocouples were connected to the recording instruments through slip rings on the cylinder shaft. A multipoint potentiometric recorder monitored the cylinder temperature and air temperature.

Drying samples were prepared from lengths of muslin cloth which were stitched two, or three, together to form a composite sandwich. Thermocouples (copper constantan) for measuring sheet temperatures were placed between these layers. The thermocouples were prepared from 0.001-in. wire to assure accurate measurement of transient temperature rise. (Experiments with thermocouples prepared from 0.010-in. and 0.005-in. wire evidenced low temperature readings due to the comparatively large mass of the thermocouples. Sheet thickness in these experiments was about 0.036 in.)

Description of Experiments

Measurements of transient temperature rise in the sheet were made by passing the wetted samples over the cylinder between a feed reel and a take-up reel while recording the signal from thermocouples in the sheet on a recording galvanometer. By passing the sample back and forth numerous times a series of recordings at decreasing moisture contents was obtained. With both channels of the galvanometer used, two recordings were obtained per pass.

A measure of moisture removed was obtained by weighing the sample at the end of each pass.

Results—Conclusions

Transient temperature measurements were not consistently reproducible from one run to another under the same conditions, nor even between two

TABLE 2. SUMMARY OF NUMERICAL SOLUTION EQUATIONS FOR CYLINDER DRYING PROBLEM

$$t = f_{(r)} \quad \tau = 0$$

$$t_{(r,\,\tau+\Delta\tau)} = \frac{t_{(r+\Delta r,\,\tau)} + t_{(r-\Delta r,\,\tau)} + \left(\frac{(\Delta r)^2}{\Delta\tau} - 2\right) t_{(r,\,\tau)}}{(\Delta r)^2/\Delta\tau}$$

Phases I and III

$$t_{(0,\,\tau)} = \frac{R_1 t_s + (1/\Delta r)\, t_{(\Delta r,\,\tau)}}{R_1 + 1/\Delta r}$$

$$t_{(1,\,\tau)} = \frac{R_2 t_a + (1/\Delta r)\, t_{(1-\Delta r,\,\tau)} - 4.65 R_2 (y_1 - y_a)\, t_b}{R_2 + 1/\Delta r}$$

$$W\tau = \sum_{\Delta\tau} 4.65 R_2\, (y_1 - y_a)\, \Delta\tau$$

Phase II

$$t_{(0,\,\tau)} = \frac{R_1 t_s + (1/\Delta r)\, t_{(\Delta r,\,\tau)}}{R_1 + 1/\Delta r}$$

$$t_{(1,\,\tau)} = \frac{R_2 t_f + (1/\Delta r,)\, t_{(1-\Delta r,\,\tau)} - 4.65 R_2 (y_1 - y_f)\, t_b}{R_2 + 1/\Delta r}$$

$$W\tau = \sum_{\Delta\tau} 4.65 R_2\, (y_1 - y_f)\, \Delta\tau$$

Phase IV

$$t_{(0,\,\tau)} = \frac{R_1 t_a + (1/\Delta r)\, t_{(\Delta r,\,\tau)} - 4.65 R_1 (y_0 - y_a)}{R_1 + 1/\Delta r}$$

$$t_{(1,\,\tau)} = \frac{R_2 t_a + (1/\Delta r)\, t_{(1-\Delta r,\,\tau)} - 4.65 R_2 (y_1 - y_a)}{R_2 + 1/\Delta r}$$

$$W\tau = \sum_{\Delta\tau} 4.65\, [R_1 (y_0 - y_a) + R_2 (y_1 - y_a)]\, \Delta\tau$$

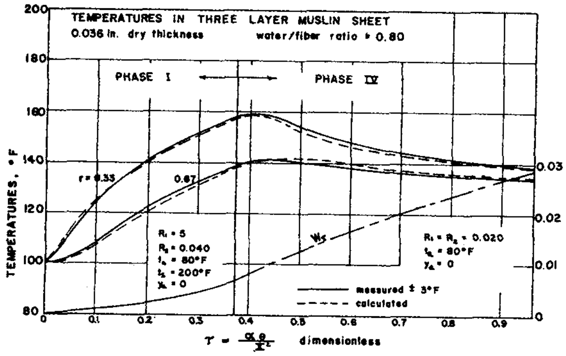

Fig. 5. Comparison of measured and calculated temperatures.

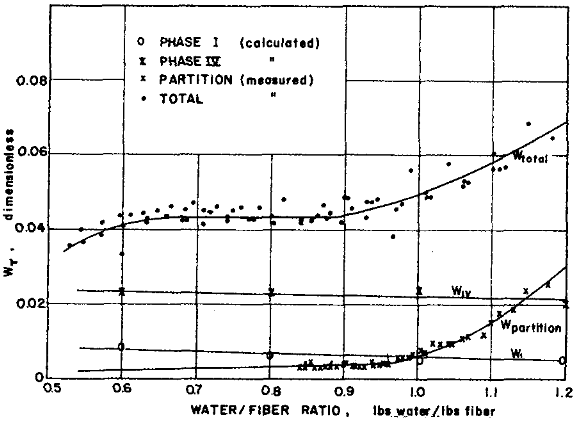

Fig. 6. Analysis of water removal.

recordings made during the same run. Differences in corresponding temperature recordings were mostly of the order of a few degrees, but in some sets the differences were as large as 15°F. In view of these variations it became desirable to define mean behavior. This was done by running replicate experiments and averaging the results. Average temperature data obtained in this way were statistically analyzed for differences between locations in the sample sheet and thermocouples used. No evidence of such differences was found. It was concluded that these differences were not due to any sample inhomogeneity but were more probably due to nonuniform wetting of (contact with) the cylinder surface and to unequal distribution of sheet tension.

Figure 4 is a graph comparing some experimentally measured (average) temperatures and some temperatures calculated from the theory given previously. In making this comparison it was possible to measure or calculate directly the value of all parameters in the theoretical calculation, except for R_1 in phase I which depends on the unknown heat transfer coefficient between cylinder surface and sheet. R_1 was then chosen to give the best fit or agreement of theory and experiment. Note however that while this one parameter was arbitrary, once a value was chosen to fit one curve at one location it applied to the calculation of another temperature curve obtained at another depth of the sheet. The average measured temperature on this and the succeeding figure was reproducible to ± 3°F.

Figure 5 is a graph similar to Figure 4 but for a somewhat dryer sheet. The excellent agreement of theory and experiment seen in these two graphs was consistently obtained in the experimental work on sheets of different moisture content and different thicknesses. In all, eight sets similar to Figures 4 and 5 were determined and observed. The results of measurements of removal of water on the dryer are given, graphically, in Figure 6. Four curves are given, representing the quantity of water removed by each of three mechanisms, and the total moisture removed. One line, marked W_I, is the quantity of water evaporated in phase I, while the sheet is on the cylinder. This quantity was calculated as part of the numerical solution for the temperatures. A second curve represents water removed by a partition between sheet and cylinder. This curve was determined by weight-loss measurements on a sample run over a cold cylinder. A third curve, W_{IV}, represents water evaporated in phase IV or between the cylinder and take-up reel on the experimental machine. This quantity was also calculated as part of the numerical solution for temperature. The uppermost curve represents total water removed, as measured during the experiments for transient temperature recording. Note that all four quantities plotted in Figure 6 were independently determined, either measured or calculated. It is seen that the three lower curves add up to 80 to 85% of the measured total. The difference may be attributed to two sources: 1. Partition was measured on a cold cylinder. It is suspected that partition may have been greater on the hot cylinder during experiments. 2. Losses were incurred while handling samples during experiments. Despite this difference the general consistency of the moisture-removal results supports the validity of the theory for temperature calculation, since these calculations were involved in the final moisture-loss calculations.

Two items of particular interest seen from Figure 6 are that the quantity of water removed by partition was of the same order as that evaporated in phase I and that most of the water removed (40 to 50%) was evaporated in phase IV. The length of phase IV, on the experimental machine, was about 1½ times the length of phase I (2.5 sec. and 1.7 sec. respectively).

This work is now being continued with experimental studies of heat transfer on a felted cylinder. It is also planned to do a more detailed study of the partition of water between sheet and cylinder. When this is done, the validity and usefulness of the theory for analysis of cylinder dryer operations of all kinds will have been demonstrated.

CONCLUSIONS

The ability of the theory to describe quantitatively and accurately the heat transfer and evaporation of water in phases I, III, and IV has been demonstrated. (Although by definition there was no phase III on the dryer as operated in these experiments, there is no real difference in the character of phases I and III.) The possible usefulness of the theory has been demonstrated in the analysis of water removal by different mechanisms.

NOTATION

Units may be chosen in any consistent system.

c = specific heat
$f_{(x)}$ = initial temperature distribution across thickness of sheet
$f_{(r)}$ = initial temperature distribution across thickness (in dimensionless length units) of sheet
H_1 = heat transfer coefficient between cylinder and sheet
H_2 = heat transfer coefficient between outer sheet surface and air
H_{2m} = mass transfer coefficient between outer sheet surface and air
k = thermal conductivity
n = integer
Q = total heat flux across a surface
q_c = convection heat flux
q_e = evaporation heat flux
R_i = dimensionless heat transfer coefficient = H_iX/k
r = dimensionless distance = x/X
t = temperature
t_b = boiling point of water

t_f = felt surface temperature
V = velocity of sheet in direction z
W_T = dimensionless evaporation quantity = $\alpha \lambda w / X k t_b$
w = quantity evaporated
X = thickness of sheet
x = distance through thickness of sheet
y = coordinate direction or distance in cross direction
y_a = mole fraction water vapor in air
y_0, y_t = mole fraction water vapor in air saturated at surface temperature of sheet
z = distance in direction of sheet travel

Greek Letters

α = thermal diffusivity = $k/\rho c$
θ = time
λ = latent heat of evaporation
τ = dimensionless time parameter = $\alpha\theta/X^2$
ρ = density

Subscripts

a = air
s = cylinder surface
f = felt

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Second Virial Coefficients of the Acetonitrile-Acetaldehyde System

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The low-pressure compressibilities of acetonitrile, acetaldehyde, and their mixtures were measured in the temperature range 40° to 100°C. From these data second virial coefficients were determined for the pure components and for the mixture. The absolute value of the coefficient B_{12}, characteristic of interaction between dissimilar molecules, was considerably larger than that for B_{11} or B_{22}, which are characteristic of interactions between similar molecules. As a result it was not possible to predict the properties of the mixture with only data for the pure components. Calculations based on the Stockmayer potential and with conventional mixing rules gave seriously incorrect results for the second virial coefficient B_{12}.

The data were interpreted with the aid of an association theory. Dimerization equilibrium constants and enthalpies and entropies of formation were computed for the pure-component dimers and for the complex. The heat of formation for the complex was significantly larger than that for the dimers; comparison with energy terms calculated from electrostatic theory suggest that the structure of the complex is different from that of the pure-component dimers.

Volumetric properties of gases and gas mixtures are of practical interest, since they provide the basic data required for the calculation of thermodynamic functions such as may be required in separations or other chemical engineering operations. In addition such properties are a valuable source of information in obtaining quantitative data on intermolecular forces. While many experimental studies have been made on the properties of nonpolar gases and their mixtures, limited information is available for polar gases, and studies on gas mixtures containing two or more polar components are very scarce. It is these mixtures however which because of strong intermolecular forces exhibit very large deviations from ideal behavior. These deviations may be sufficiently high to invalidate seriously the ideal-gas assumption even at rather low pressures. In addition the nature of polar-polar interactions is often so specific that the forces between two dissimilar molecules is no simple average of the forces acting between molecules of the same species; in polar gas mixtures therefore the Lewis fugacity rule is usually most unreliable. As a contribution to understanding the thermodynamic properties of polar substances this work presents results on the volumetric behavior of two highly polar gases and their mixtures. Data are reported on the low-pressure compressibility of the acetoni-

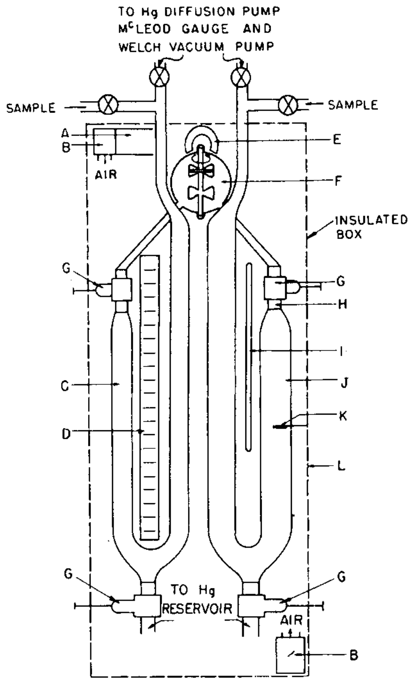

Fig. 1. *P-V-T-y* apparatus: *A* = heater duct, *B* = circulation blower, *C* = 0.50000-in. U tube, *D* = standard meter bar, *E* = magnet, *F* = mixing chamber, *G* = hoke bellows valve, *H* = Kovar-pyrex graded glass seal, *I* = thermometer, *J* = 0.75000-in. U tube, *K* = thermistor.

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